Persistent homology and partial matching of shapes

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Abstract The ability to perform not only global matching but also partial matching is investigated in computer vision and computer graphics in order to evaluate the performance of shape descriptors. In my talk I will consider the persistent homology shape descriptor, and I will illustrate some results about persistence diagrams of occluded shapes and partial shapes. The main tool is a Mayer-Vietoris formula for persistent homology. Theoretical results indicate that persistence diagrams are able to detect a partial matching between shapes by showing a common subset of points both in the one-dimensional and the multi-dimensional setting. Experiments will be presented which outline the potential of the proposed approach in recognition tasks in the presence of partial information.

Keywords Mayer-Vietoris formula; persistence diagrams; shape occlusions; sub-part detection.

1 Introduction

Geometrical-topological methods for shape description and comparison are increasingly studied in computer vision, computer graphics and pattern recognition [1, 4, 7, 8]. The common idea underlying the methods of this class is to perform a topological exploration of the shape according to some quantitative geometric properties provided by a real function chosen to extract shape features. For this reason they could be called shape-from-function methods. The attractive feature of shape-from-function techniques is that they concisely capture shape information, in a manner that can be robust to deformation while being able to cope with changes in object viewpoint at a multiresolution level [3]. In this context, persistent homology groups have been developed to analyze and compare multidimensional shapes, with application mainly to 2D digital shapes equipped with scalar functions [9, 15, 18] and 3D digital shapes (represented by surface or volume models) equipped with vector-valued functions [2].

A common requirement for shape descriptors is the ability to perform not only global matching, but also partial matching. Therefore, although many shape descriptors work on a shape as a whole, the problem of how to detect similarity even when only a sub-part of one shape is occluded is a widely researched topic.

In my talk I will address the partial shape matching problem for persistent homology groups. In particular I will illustrate results proved in [11] on the behavior of persistent homology groups in the presence of shape occlusions, i.e. when foreground objects overlap the object under investigation hiding significant portions of one shape.

Recent results showing that also multidimensional persistent homology shares the same ability to preserve not only global, but also local information will also be presented together with experiments illustrating how multidimensional persistent homology can be used to deal with the sub-part detection problem of 3D models.

2 Persistent homology groups: a shape descriptor

Given an object X, which we assume to be a triangulable space, its shape can be studied through measurements on its points represented by a function $f : X \to \mathbb{R}$. For example, one could measure curvature, torsion, distance from the center of mass, height etc. at each point. Now the properties of (X, f) can be analyzed through the topological changes (e.g. creation or annihilation of connected components, tunnels, voids) occurring in the lower level sets $X_u = \{p \in X : f(p) \leq u\}$ as u varies in $\mathbb{R}$. Using algebraic topology, these topological changes in the spaces X_u can be detected computing the homology groups $H_*(X_u)$. More precisely, when $u < v$, we can consider the homomorphism $H_*(X_u) \to H_*(X_v)$ induced by the inclusion of X_u in X_v. The kernel of this homomorphism gives us information concerning the death of homology classes of cycle in passing from X_u to X_v, while its image tells us something about the classes that survive. Exactly this image is nowadays known as the *persistent homology group* of (X, f) at (u, v), denoted $H_*^{u,v}(X, f)$. Thus, the knowledge of all the groups $H_*^{u,v}(X, f)$, varying $(u, v) \in \mathbb{R}^2$ with $u < v$, gives us an insight into the topological features of X as seen through f, turning it out into a shape descriptor.

3 A Mayer-Vietoris formula for persistent homology

In algebraic topology it is well-known that, using the Mayer-Vietoris sequence, the homology of a space X can be studied splitting X into subspaces A and B, and computing the homology of A, B, $A \cap B$. A natural question is to which an extent persistent homology benefits of a similar property. In [11] it was shown that persistent homology has a Mayer-Vietoris sequence that in general is not exact but only of order two. However, a Mayer-Vietoris formula involving the ranks of the persistent homology groups of X, A, B and $A \cap B$ plus three extra terms was obtained. This implies that topological features of A and B either survive as topological features of X or are hidden in $A \cap B$.

4 Persistence diagrams and shape recognition in the presence of occlusions

The information contained in all the groups $H_*^{u,v}(X, f)$, varying $(u, v) \in \mathbb{R}^2$ with $u < v$, turns out to be redundant. The same information can be summarized by a discrete representation through the so-called *persistence diagrams*. These are countable collections of points in the plane with multiplicity. Each of these points represents the pairing between the birth of a homology class and its death along the filtration of X given by the lower level sets of f.

Given $X = A \cup B$, our Mayer-Vietoris formula for persistent homology groups implies that points of the persistence diagrams of A and B leave a fingerprint in the persistence diagram of X. Indeed, persistence diagrams are able to detect a partial matching between two shapes by showing a common subset of points. In this way it is shown that persistent homology groups are able to preserve local information.

As an application of partial matching detection, we consider the problem of shape recognition in the presence of occlusions. Basically, the interest in robustness against partial occlusions is motivated by the problem of recognizing an object partially hidden by some other foreground object in the same image. The ability of recognizing shapes, even when they are partially occluded by another pattern, has been investigated in the Computer Vision literature by various authors, with reference to a variety of shape recognition methods (see, e.g., [10, 12, 13, 14, 16, 17]).

Our experiments on recognition of occluded shapes via persistence diagrams show that persistent homology can manage uncertainty due to the lack of complete information.

5 Multidimensional persistent homology and partial matching

Multidimensional persistent homology groups are the extension of the notion of persistent homology group to the case when a multi-filtration instead of a filtration is used to grow the studied space. Multi-filtrations of a space X can be obtained by considering lower level sets of appropriate vector-valued functions $f = (f_1, \ldots, f_n)$ defined on X.

The idea of using multi-filtrations arises from the observation that the shape of an object can be more thoroughly characterized by means of a set of properties, each investigating specific features of the shape under study, e.g., scientific data from physical or medical studies that typically consist of a large number of measurements taken within a domain of interest.

Multidimensional persistent homology groups do not admit a complete discrete representation analogous to persistence diagrams for one-dimensonal presistent homology [6]. Therefore, even if it is straightforward to obtain a Mayer-Vietoris formula also in this case using the same arguments as in the one-dimensional setting, detecting a partial matching in multidimensional persistent homology is not straightforward.

In this talk I will show that, using the foliation method developed in [5] in order to carry out the one-dimensional reduction of multidimensional persistent homology, we can still assess a partial matching, as was recently proved.

Examples on 3D models analyzed through vector-valued functions will be presented.

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