Characterizing Configurations of critical points through LBP Extended Abstract

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***Abstract*—In this abstract we extend ideas and results submitted to [3] in which a new codification of Local Binary Patterns (LBP) is given using combinatorial maps and a method for obtaining a representative LBP image is developed based on merging regions and Minimum Contrast Algorithm. The LBP code characterizes the topological category (max, min, slope, saddle) of the 2D gray level landscape around the center region. We extend the result studying how to merge non-singular slopes with one of its neighbors and how to extend the results to non-well formed images/maps. Some ideas related to robust LBP and isolines are also given in last section.**

***Keywords*—ocal binary patterns, critical points, combinatorial mapsocal binary patterns, critical points, combinatorial mapsl**

I. LBP CODES AND COMBINATORIAL MAPS

Given a grayscale digital image I, the local-binary-pattern codification of I, $LBP(I)$ [8], [9] is a grayscale digital image (LBP codes) used to represent the texture element at each pixel in I.

In this paper, for computing LBP codification, the 4 neighbors (on its top, bottom, right, left) of each pixel are considered for comparison. Where the center pixel's gray value is smaller than the neighbor's gray value, write 1. Otherwise, write 0. Example:

$$\begin{array}{|c|c|c|}\hline 113 & 240 & 23 \\ \hline 20 & 25 & 12 \\ \hline 15 & 30 & 40 \\ \hline \end{array} \Rightarrow \quad \begin{array}{ccc} & 1 & \\ 0 & 25 & \mathbf{0} \\ & 1 & \end{array} \quad \Rightarrow \mathbf{0}101 \Rightarrow 9$$

A combinatorial map in two dimensions (shortly cal led $2-$maps) [1], [7] consists of the triplet $G = (\boldsymbol{\mathcal{D}}, \sigma, \alpha)$, where $\boldsymbol{\mathcal{D}}$ is a set called the set of darts and σ, α are two permutations defined on $\boldsymbol{\mathcal{D}}$ such that α is an involution: $\forall d \in \boldsymbol{\mathcal{D}} \quad \alpha^2(d) = d$. The composition $\sigma\alpha$ is denoted by φ. Each dart $d \in \boldsymbol{\mathcal{D}}$ defines a region by the orbit[1] of φ:

$$\varphi^*(d) = \{\varphi(d), \varphi^2(d), \ldots, \varphi^n(d) = d\}.$$

The vertices of the region are given by the set $\sigma^*(d)$. Two regions $\varphi^*(d_1)$ and $\varphi^*(d_2) \neq \varphi^*(d_1)$ are *direct neighbors* if their orbits have a non-empty intersection:

$$\varphi^*(d_1) \cap \varphi^*(d_2) \neq \emptyset.$$

They are *corner neighbors* if they are not direct neighbors but there exist $d'_i \in \varphi^*(d_i)$, $i = 1, 2$ such that $d'_2 \in \sigma^*(d'_1)$.

Two basic operations, *edge removal* and *edge contraction* can simplify the potentially huge[2] $2-$map to obtain a simpler subdivision of the object in terms of darts (for details see [5], [1]). A *combinatorial pyramid* [2] is a stack of $2-$maps created by successive removal and contraction operations.

In our case, pixels are considered unit-square regions. $2-$maps are computed for these regions. The intensity of a pixel/region p is denoted by $g(p)$. For each dart d, we assign a binary value $\ell(d) \in \{0, 1\}$ which coincides with the LBP condition. That is, suppose dart d starts at the center pixel p and is $\alpha-$related to a neighbor pixel n (i.e., n is a direct neighbor of p). If the center pixel's gray value is smaller than the neighbor's gray value, then $\ell(d) = 1$. Otherwise, $\ell(d) = 0$. For a sequence of darts $S = (d_1 d_2 d_3 \ldots)$, we denote by $\ell(S)$ the corresponding sequence of binary labels $\ell(S) = (\ell(d_1)\ell(d_2)\ell(d_3)\ldots)$.

The LBP code characterizes the *topological category* of the gray level landscape around the center region without referring to the original gray values. A region $\varphi^*(d_r)$ is a *local maximum* if the LBP code $\ell\varphi(d) = 0 \forall d \in \varphi^*(d_r)$ contains just 0s. A *local minimum* produces an LBP code with just 1s. A region $\varphi^*(d)$ is a *plateau* if it is direct neighbor to another region with same gray value, i.e., if there exists $d' \in \varphi^*(d)$ such that $\ell(d) = \ell\alpha(d) = 1$. A region $\varphi^*(d)$ is a *slope* if it contains exactly one connected component of 1s and one connected component of 0s in the closed path of LBP bits around the region, $(11 \ldots 100 \ldots)$. Otherwise it is a *saddle*. In particular, we say that a slope is *singular* if $\ell(\varphi^*(d))$ contains exactly one 1 or one 0. It is *double-singular* if its LBP code is exactly 01 or 10.

Minimum Contrast Algorithm. In [3], the Minimum Contrast Algorithm for reconstructing a representative image R from its LBP code [11], is translated in terms of $2-$maps (see Fig. 1). Although R may be quite different from the original image, it generates identical LBP codes. The top image overlays the reconstructed representative image with some monotonic increasing paths (in red) that are the basis for the reconstruction. Since the contrast between successive pixels along these

[1] sequence of darts

[2] An $n \times m$ image needs nearly $4nm$ darts

paths must be at least one the reconstructed gray value of a pixel corresponds to the longest monotonic path to to a local minimum (marked by a green dot) which is set to zero.

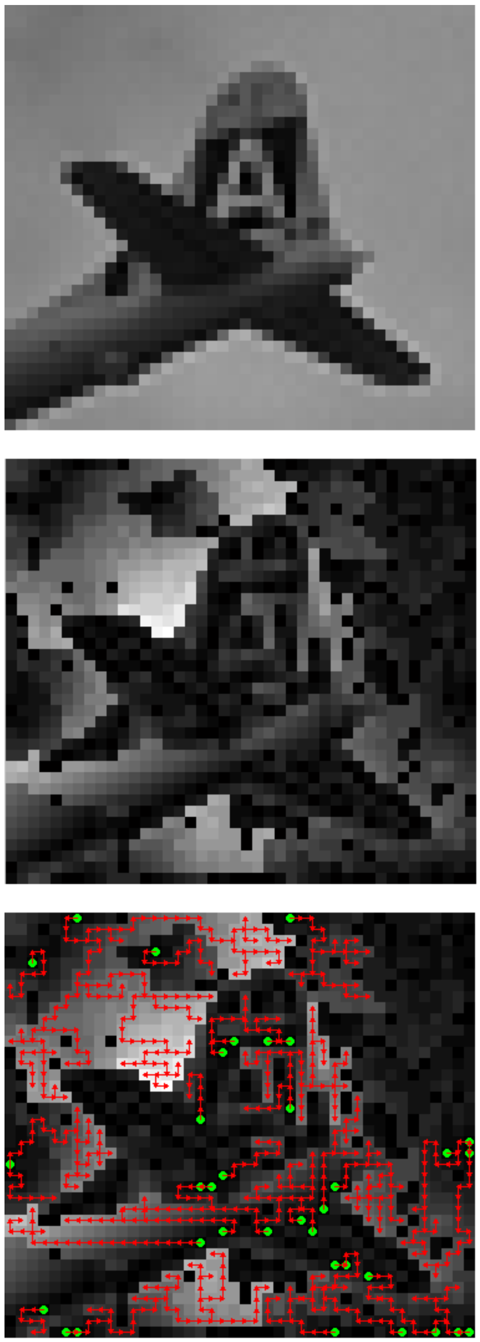


Fig. 1. Top: the original image. Center: reconstructed image. Bottom: Minimum pixels (dots) and monotonic ascending paths (lines).

Merging regions. Our final aim is to obtain a reconstructed image R and a structure consisting only of local minima, local maxima, saddles and the respective adjacency relationships (similar to Reeb graphs [10]).

In [3], a process to merge plateaus and singular slope in terms of $2-$ maps is given. Multiple edges are also merged via the contraction operation. After applying the merging process described in [3] on the top of the pyramid we have: vertices of degree 3 or 4, non-singular slopes, double-singular slopes, maxima, minima and saddles. We do not have: multiple edges (vertices of degree 2), singular slopes nor plateaus. See Fig. I in which the green region is a double-singular slope.

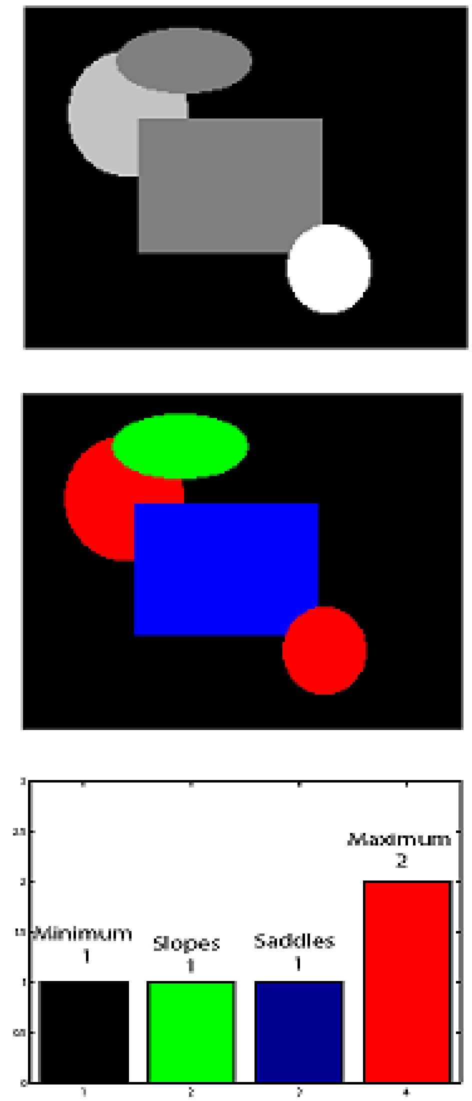


Fig. 2. At the top, the original image. The two pictures on the bottom are an illustration of the five regions obtained on the top level of the pyramid.

Our further aim here is to study how to merge the rest of the slopes with one of its neighbors.

II. To Be or Not to Be: Well-Composed Images

A 2D image is *well-composed* [6] if it (or its complement) does not contain the following configuration:

a	b
c	d

where $g(a) < g(b)$, $g(a) < g(c)$, $g(d) < g(b)$ and $g(d) < g(c)$. For example, the following block of 2×2 pixels cannot be part of a well-composed image:

12	117
191	14

The notion of well-composed configurations can be extended to regions.

Lemma 1: [3] If the image is well-composed and there are no plateaus, then $4-$ and $8-$connectivity LBP provide pixels/regions with equivalent topological categories (i.e. same pixels/regions are labeled with max, min, slopes or saddles from both).

In order to merge slopes, our strategy could be to merge regions with minimum (resp. greatest) intensity value around a minimum (resp. maximum). Nevertheless, we have the following result.

Lemma 2: Let d_m be a dart in a region m. Let $\varphi^*(d_m)$ be the surrounding darts. $N(m) = \varphi^*(\sigma^*(\varphi^*(d_m))) \setminus \varphi^*(d_m)$ form a closed cycle of regions surrounding m, and consist of all the direct and corner neighbors of m. Consider the pyramid obtained after merging plateaus, singular slopes and multiple edges. Let m be a minimum (resp. maximum) and let n be a region with minimum (resp. maximum) intensity value in $N(m)$.

- If n is a direct neighbor of m and $|\varphi^*(d_n)| > 2$ then n is a saddle.
- If the image is well-composed then n cannot be a corner neighbor of n.

Proof: To prove the result, suppose m is a minimum and n a direct neighbor of m. Observe that n cannot be a minimum nor a maximum if there are other regions in $N(m)$. Since n is a region with minimum intensity value in $N(m)$ and $|\varphi^*(d_n)| > 2$ then the sequence 101 appears in the LBP code of n. Observe that $|\varphi^*(d_n)| > 3$, otherwise n would be a singular slope which is not possible. For the same reason (n is not a singular slope), LBP code of n should contain more than one 0. Therefore, n is a saddle. If n is a corner neighbor of m then a non-well formed configuration appears on the corner. ■

In the following result we show a special case in which a slope can be merged with one of its direct neighbors.

Lemma 3: Consider the pyramid obtained after merging plateaus, singular slopes and multiple edges. Let $a = \varphi^*(d)$ be a (non double-singular) slope. Let $b = \varphi^*(\alpha(d))$ (i.e., b is a direct neighbor of a). If

- $\ell(d) = 0$, (resp. $\ell(d) = 1$),
- $g(b) > \max\{g(n) : n$ is a direct neighbor of a and $g(a) > g(n)\}$ (resp. $g(b) < \min\{g(n) : n$ is a direct neighbor of a and $g(a) < g(n)\}$)
- and $(\ell\sigma(d), \ell\sigma\alpha(d)) = (0,1)$ or $(\ell\sigma^{-1}(d), \ell\sigma^{-1}\alpha(d)) = (0,1)$

then a can be merged with b and the new region $a \cup b$ has the same topological category and intensity value than b.

Proof: Suppose that $\ell(d) = 0$ and $(\ell\sigma(d), \ell\sigma\alpha(d)) = (0,1)$. This means that $g(a) > g(b)$. The code of the new region consists of replacing the bit $\ell\alpha(d) = 1$ in the LBP code of b by the string $0\ldots01\ldots1$ (see Fig.II.bottom). Since $g(b) > \max\{g(n) : n$ is a direct neighbor of a and $g(a) > g(n)\}$ then the code of the merged region is in fact its LBP code. Finally, since $\ell\sigma\alpha(d) = 1$, then the bit $\ell\alpha(d) = 1$ in the LBP code of b is preceded by 0 therefore, the topological category of the new region is the same than that of b. ■

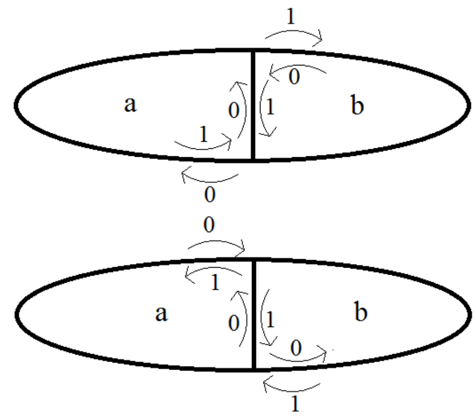


Fig. 3. Top: Illustration of condition $(\ell\sigma(d), \ell\sigma\alpha(d)), \ell\sigma\alpha(d)) = (0,1)$. Bottom: Illustration of condition $(\ell\sigma^{-1}(d), \ell\sigma^{-1}\alpha(d)) = (0,1)$.

Non-well-composed Images. If the image is not well-composed, the idea is to detect the 2×2 blocks corresponding to saddles on the dual graph.

Lemma 4: Merging pixels in a non-well-composed configuration changes one of their topological categories.

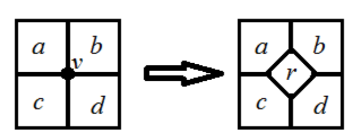
Proof: Suppose we detect a non-well-composed configuration, i.e., a 2×2 block:

a	b
c	d

such that $g(a) < g(b)$, $g(a) < g(c)$, $g(d) < g(b)$ and $g(d) < g(c)$. Suppose the LBP code of b is 0000 (i.e. b is a maximum). Suppose a is a slope. If b merges with a, the new region cannot be a maximum since the code of the new region contains at least one 1 (the one that comes from the inequality $g(c) < g(a)$. Although the merged region can be a slope it destroys the maximum b. Two adjacent regions cannot be both slopes. And none of the four regions can be a saddle since the two adjacent regions have both either a larger or a smaller gray value. Other cases are similar. ■

Therefore, if the image is not well-composed then slopes can survive, after merging-slope process, until the top of the pyramid.

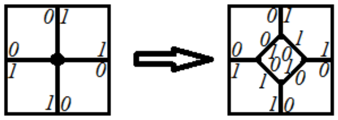
In order to avoid this, the idea is to replace vertex v shared by the four pixels a, b, c, d by a new region r with a new gray scale value $g(r)$ reflecting the relations between a, b, c, d:



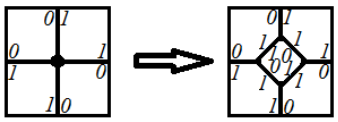
Without loss of generality, we can suppose that $g(a) \leq g(d) < g(b) \leq g(c)$. Following cases can occur:

1) $g(a) = g(d) < g(b) = g(c)$ (resp. $g(a) < g(d) < g(b) < g(c)$). In this case, the vertex v

represents a saddle. Set $g(r) = \frac{g(d)+g(b)}{2}$. We have that $g(a) = g(d) < \frac{g(b)+g(d)}{2} < g(b) = g(c)$ (resp. $g(a) < g(d) < \frac{g(b)+g(d)}{2} < g(b) < g(c)$). Therefore, the LBP code of r is 0101 which is a saddle and all three new vertices have degree 3 and, hence, are well-composed.



2) $g(a) = g(d) < g(b) < g(c)$ (resp. $g(a) < g(d) < g(b) = g(c)$). Set $g(r) = g(d)$ (reps. $g(r) = g(b)$). In our process, regions $g(a) = g(r) = g(d)$ (resp. $g(b) = g(r) = g(c)$) form a plateau.



III. Robust LBP and Isolines

In this last section we would like to address a few issues related to the above research that will be part of our future research.

Since we know the contrast $|g_{max} - g_{min}|$ between end points of monotonic paths in the original image and in the minimum contrast representative image (=length of monotonic path), the average step along a path measures by how much we have to change a gray value along this path in order to change the LBP output. An average contrast of 1 would make the result extremely sensible to small changes, either creating a new plateau or a new critical point. The result could be made more robust by simply merging instable sub-configurations (e.g. darts with low contrast, similar to robust local binary pattern, RLBP).

Another idea concerns isolines (also called level sets, curves of constant height) emanating from saddle points: isolines can split and merge only at saddle points. Hence multiple splits and merges can occur only in degenerated cases where saddles have identical height ('all distinct critical points must have different function values', Morse theory). Hence a non-degenerate saddle of degree 4 must create two iso-cycles both surrounding either the neighbor's max or min. If the cycles surround a max then the two min regions are a single connected region since you can walk around the two max keeping one hand on the isoline. However we may find degenerated cases in our discrete data. Does this happen in our data? A way to solve such discretization problems [4] is to slightly incline the heights of such critical points from one side of the domain to the other such that the difference is always less than 1, i.e. adding i/m to each pixel (i, j).

If isolines/level sets through a non-degenerated saddle point are homology generators, we could consider the persistence of the enclosed substructures and probably eliminate less persistent parts (even if they are critical). It seems that many of our images contain lots of noisy critical points with small persistence. So far we considered only simplifications that preserve critical points and the LBP codes. In order to simplify the result we could also select the least persistent structures for simplification. We could consider a similar strategy as for the re-insertion and de-contraction of darts: with a little extra information we could reconstruct the lower pyramid level if needed. Is the LBP-label of the eliminated dart sufficient to reconstruct the finer resolution?

Furthermore we can consider reconstruction at different levels:

a) reconstruct only the original LBP codes (without considering whether the original image belongs to the reconstruction class);

b) reconstruct the min contrast representative and determine the size of the class of images it represents; and

c) reconstruct the original image: under what conditions has the class only one representative.

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