Massive CO+binatorics

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Problemset

- **(Quant Guide)** Suppose you are continually rolling a standard fair 6-sided die. Find the probability that all of the odd values show up before the first even value.

- **(Conde's Archive)** How many subsets of the set $\{1, 2, \ldots, n\}$ satisfy the condition of containing no two consecutive integers?

- **(2006 AIME)** Let S be the set of real numbers that can be represented as repeating decimals of the form $0.\overline{abc}$ where a, b, c are distinct digits. Find the sum of the elements of S.

- **(Online Math Open 2014)** Kevin has $2^n - 1$ cookies, each labeled with a unique nonempty subset of $\{1, 2, \ldots, n\}$. Each day, he chooses one cookie uniformly at random out of the cookies not yet eaten. Then, he eats that cookie, and all remaining cookies that are labeled with a subset of that cookie. Determine the expected value of the number of days that Kevin eats a cookie before all cookies are gone.

- **(LeetCode Weekly Contest)** Let n be a positive integer. We wish to tile a $2 \times n$ board using two types of pieces:

  - Dominoes of size 2×1 (or 1×2).
  - L-shaped trominoes, formed by removing one unit square from a 2×2 square.

  The pieces may be rotated freely. The tiling must cover the board completely with no overlaps. Let $f(n)$ denote the number of such valid tilings. Determine a recurrence relation for $f(n)$.

- **(IMO 2001)** Let $n > 1$ be an odd integer and let $c_1, c_2, \ldots, c_n$ be integers. For each permutation $\pi = (\pi_1, \pi_2, \ldots, \pi_n)$ of $\{1, 2, \ldots, n\}$, define $S(\pi) = \sum_{i=1}^{n} c_i \pi_i$. Prove that there exist two permutations $\pi \neq \mu$ of $\{1, 2, \ldots, n\}$ such that $n!$ is a divisor of $S(\pi) - S(\mu)$.

- **(MOP Test 2007)** In an $n^2 \times n^2$ array, each of the numbers $1, 2, \ldots, n^2$ appears exactly n^2 times. Show that there is a row or a column in the array with at least n distinct numbers.

- **(USAMO 2018)** Let p be a prime, and let $a_1, \ldots, a_p$ be integers. Show that there exists an integer k such that the numbers
  $$a_1 + k, \quad a_2 + 2k, \quad \ldots, \quad a_p + pk$$
  produce at least $\frac{1}{2}p$ distinct remainders upon division by p.

- **(Czech-Polish Junior 2019)** In a group of children, every child has exactly d friends, and every pair of children who are not friends share exactly one common friend. Prove that there are at most $d^2 + 1$ children in the group.