Transition phase in biological systems

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We introduce some discrete models, each one with a greater order of complexity, to study the emergence of motion in systems of j particles ($j = 1...N$) with biological motivated interaction. By decreasing noise, is possible to present numerical evidence that the system shows a kinetic phase transition from a disordered into a ordered state. First model finds that this phase transition is continuous. Adding one term to this model is observed that it is discontinuous. Third model, more oriented to cellular dynamic, finds that the system exhibits a continuous transition.

Vicsek et al.(1995)

Velocity of a particle is constructed to have an absolute value v_0 and a direction given by the angle $\theta_j(t+1)$;

$$v_j(t+1) = v_0 e^{i\theta_j(t+1)} \quad (1)$$

$$\theta_j(t+1) = arg\left(\sum_{j=1}^{N} e^{i\theta_j(t)}\right) + \eta\xi_j(t) \quad (2)$$

where $arg\left(\sum_{j=1}^{N} e^{i\theta_j(t)}\right)$ denotes the average direction of the particle velocities being within a circle of radius r surrounding the given particle. Second term represents noise. This introduce a tendency to align with neighbors. In the absence of noise, interacting particles align perfectly. For maximal noise particles describe random walks. This transition is characterized instantaneous by the following order parameter:

$$\psi(t) = \frac{1}{Nv_0}\left|\sum_{j=1}^{N} v_j(t)\right| \quad (3)$$

Grégorie and Chaté(2004)

The difference with Vicsek model is noise term. Instead of having "perfect direction" it can be argued that errors are made when estimating the interactions,

$$\theta_j(t+1) = arg\left(\sum_{j=1}^{N} e^{i\theta_j(t)} + \eta n_j(t) e^{i\xi_j(t)}\right) \quad (4)$$

where $n_j(t)$ denotes the number of neighbors of particle j in time t. In this case of varying noise it can be seen that transition is discontinuous. It is showed in Fig. 2.

Szabó et al.(2006)

Assuming that intercellular forces through which particles interact are considered to be short-range interactions. Thus, velocity for particle j is given by:

$$\mathbf{v}_j(t) = v_0\mathbf{n}_i(t) + \mu\sum_{i=1}^{N} \mathbf{F}(\mathbf{r}_i, \mathbf{r}_j) \quad (5)$$

where $n_i(t)$ denotes the direction vector and μ denotes strength of interaction force. The direction of self-propelling velocity $n_i(t)$, described by $\theta_j(t)$, tries to relax the system, which also experiences Gaussian noise can be approximated as:

$$\frac{d\theta_j}{dt} = -\frac{1}{\tau}\theta_j + \frac{\eta}{\sqrt{12}}\xi(t) \quad (6)$$

where τ is the relaxation time. In this model, parameter order is given by the following formula

$$\bar{V} = <\frac{1}{N}\left|\sum_{j=1}^{N}\frac{v_j(t)}{|v_j(t)|}\right|> \quad (7)$$

Comparison between models

Introducing angular noise, the behaviour of the order parameter jumps smoothly to zero when the amount of noise is reduced,

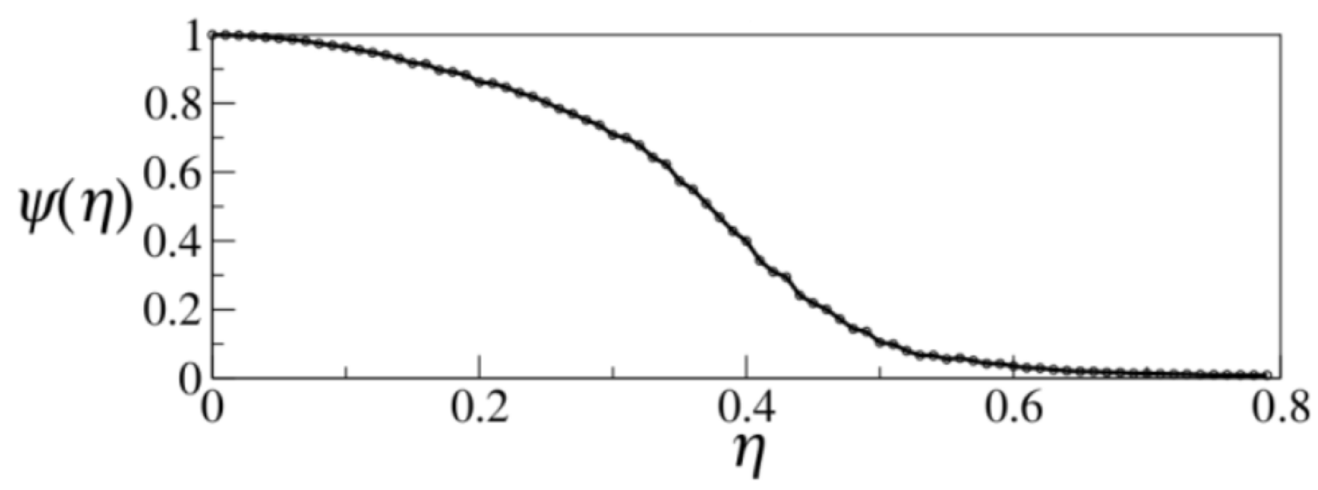

Fig. 1: Variation of average order parameter as a function of angular noise in the original Vicsek model.

whereas in the original Grégorie and Chaté model is varied abruptly,

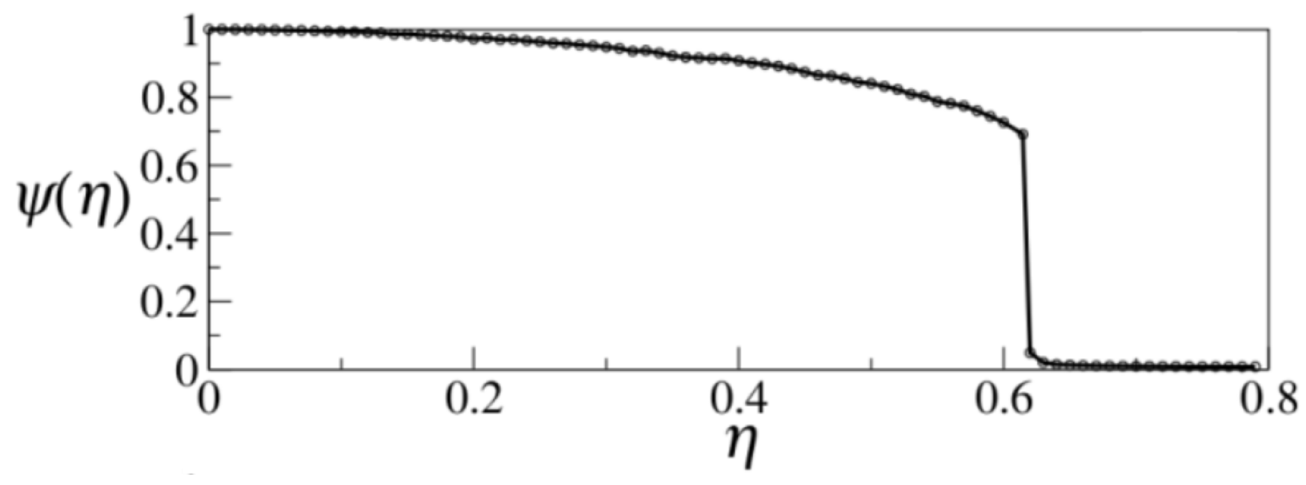

Fig. 2: Variation of average order parameter as a function of vectorial noise in the Grégorie and Chaté model.

Szabó et al.(2006) drawing an analogy between their experimental results and model, find that emergence of collective motion is a continuous phase. In contrast with [2], they argue that angular noise models define universally continuous phase transition.

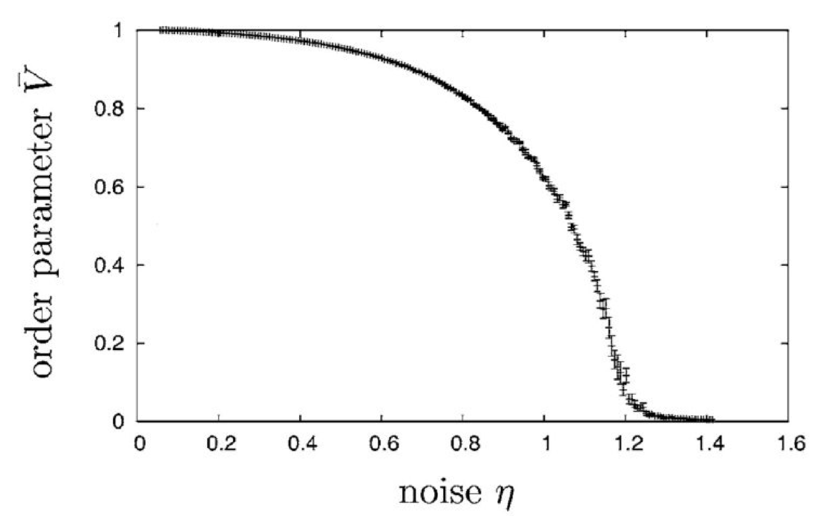

Fig. 3: The average value of $\bar{V}$ of average order parameter as a function of noise.

Conclusions

To sum up, validity of these models depend on the system studied. Model proposed by *Vicseck et al.* is a simple model which describes well enough the emergence of self-ordered motion observed in biological systems with interaction. Although *Grégorie and Chaté* demostrate that transition which takes place in the system is discontinuous, in contrast with [1]. Finally, *Szábo et al.* describe satisfactorily cellular motion adding a short-range force. As result, they find a continuous phase transition.

References

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[2] G. Grégoire, H. Chaté. *Onset of Collective and Cohesion Motion*. In: *Physical review letters*, 2004.
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