Discrete breathers in a nonlinear electrical transmission line

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Presentation of the model

- Line = 32 single cells
- Voltage source: $V(t) = V_d\, cos(\omega t)$ with $1V \leq V_d \leq 5V$ and $200 \leq f(kHz) \leq 600$
- Nonlinear element: Varactor diode (NTE 618)

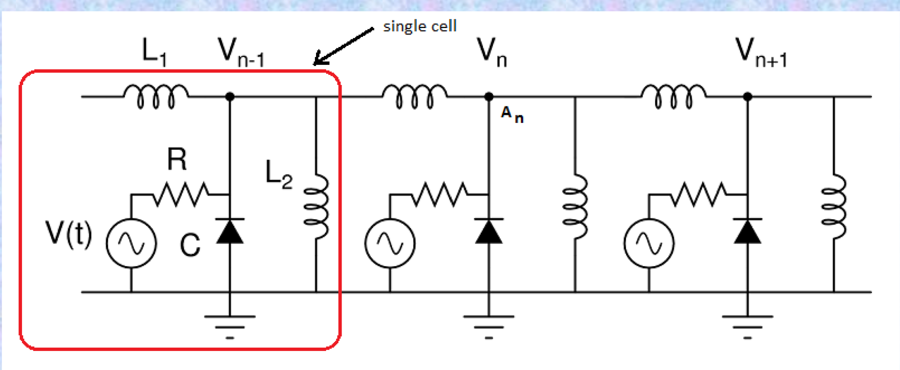

Figure 1: Sketch of the electrical transmission line

Nonlinear resonance curves of the single cell

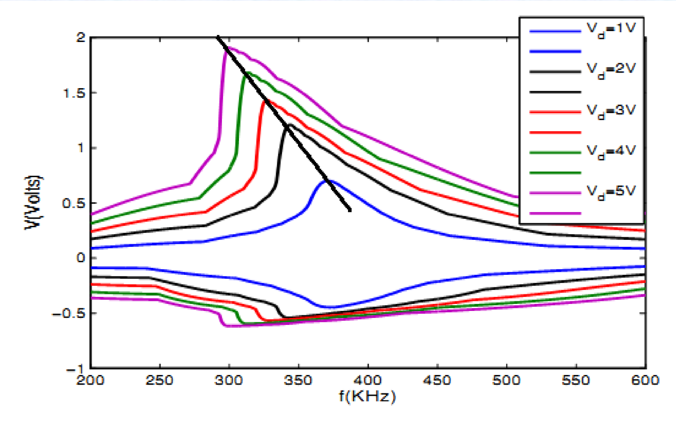

Figure 2: Nonlinear resonance curves of the single cell

Dynamical equations of the full electrical line and dispersion relation

- Dynamical equations

$$\begin{cases} C(V_n)\dfrac{dV_n}{dt} = Y_n - I_D(V_n) + \dfrac{V_d\cos(\omega t)}{R} - (\dfrac{1}{R} + \dfrac{1}{R_l})V_n \\ L_2\dfrac{dY_n}{dt} = \dfrac{L_2}{L_1}(V_{n+1} + V_{n-1} - 2V_n) - V_n \end{cases} \quad (1)$$

- Dispersion relation

$$\omega^2 = \omega_0^2 + 4u_0^2\sin^2(\frac{k}{2}) \quad (2)$$

Dispersion relation curve

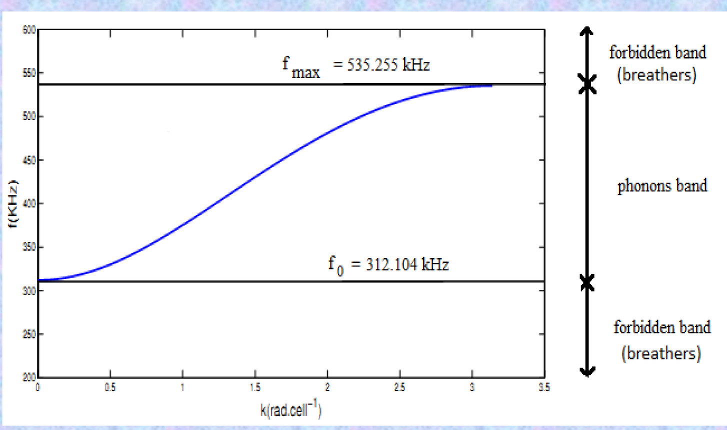

Figure 3: dispersion relation curve of the electrical line.

The Complex Cubic Ginzburg-Landau equation

- Semi-discrete approximation

$$V_n(t) = \varepsilon V_{11}(x,t)e^{i\theta(n,t)} + c.c + \varepsilon^2\left[V_{20}(x,t) + V_{22}(x,t)e^{2i\theta(n,t)} + c.c\right] \quad (3)$$

$$i\frac{\partial V_{11}}{\partial \tau} + P\frac{\partial^2 V_{11}}{\partial^2 \xi} + Q\,|V_{11}|^2 V_{11} = 0 \quad (4)$$ where $\tau = \varepsilon^2 t$ and $\xi = \varepsilon(x - V_g t)$

$$P = P_r + iP_i \quad (5) \qquad Q = Q_r + iQ_i \quad (6)$$

$$P_r = -\frac{u_0^2}{2\omega^3}(u_0^2\sin^2 k - \omega^2\cos k) \quad (7) \qquad P_i = 0 \quad (8)$$

$$Q_r = \frac{2c^2u_0^2\omega\sin^2 k}{\omega_0^2 + 4u_0^2\sin^4(\frac{k}{2})} - \frac{\left[4c^2(4(\omega^2 - u_0^2\sin^2 k) - \omega_0^2) + 8abc\right]\omega^3}{\left[4(\omega^2 - u_0^2\sin^2 k) - \omega_0^2\right]^2 + 4a^2\omega^2} + \frac{3}{2}d\omega \quad (9)$$

$$Q_i = \frac{2bcu_0^2\sin^2 k}{\omega_0^2 + 4u_0^2\sin^4(\frac{k}{2})} - \frac{\left[4bc(4(\omega^2 - u_0^2\sin^2 k) - \omega_0^2) - 8ac^2\omega^2\right]\omega^2}{\left[4(\omega^2 - u_0^2\sin^2 k) - \omega_0^2\right]^2 + 4a^2\omega^2} \quad (10)$$

Existence of breather solutions

- Condition of existence
- General condition:

$P_rQ_r + P_iQ_i > 0$ (11)

(modulational instability condition)

- Our particular case

$P_i = 0$

$P_rQ_r > 0$ (12)

for k < 1.31 , the modulational instability condition is satisfied, and CCGL admits breathers solutions

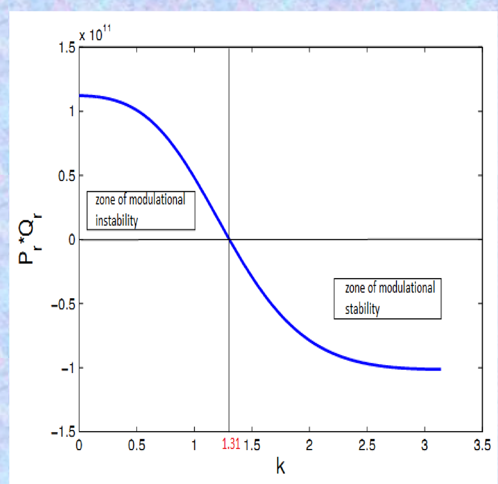

Figure 4: Variations of the PrQr product as a function of the wave vector k

Results of numerical simulations

- Observation of the modulational instability and formation of a stationary multibreather

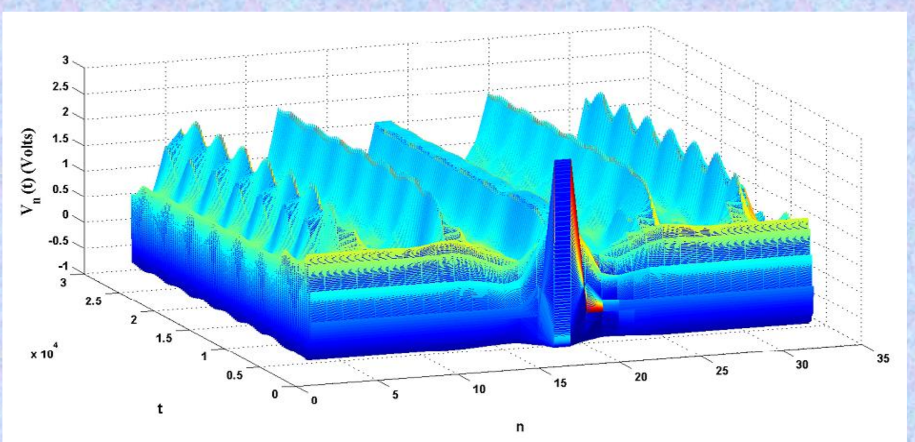

Figure 5: time evolution of a multibreather generated by modulational instability for V_d = 4.5 V and $f = 285\ kHz$

Results of numerical simulations

- Spatial profile of the mulbreather

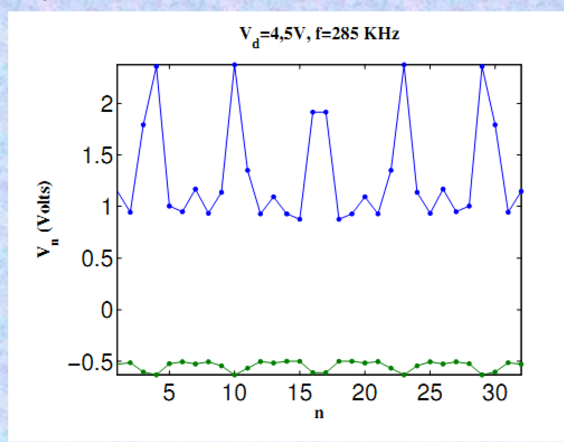

Figure 6: spatial profile of the multibreather at two differents times

Conclusions

- **We have shown that the propagation of a wave packet in the System can be modeled by the Complex Cubic Ginzburg-Landau (CCGL) equation and that this equation admits breathers solutions when the modulational instability condition is satistied,**
- **We have shown numerically how the modulational instability leads to the formation of a stationary multibreather.**

References

- F. Palmero, L.Q. English, J. Cuevas, R. Carretero-Gonzalez, P.G. Kevrekidis, Discrete breathers in a nonlinear electric line: Modeling, computation, and experiment, Phys. Rev. E 84 (2011) 026605

- Tsoplefack Joël François, Master Thesis: Discrete breathers in a dissipative bi-inductance nonlinear electrical line, Tchawoua Clément, supervisor. Université de Yaoundé I (2013).