Open Problem

ABOUT THE INTEGRABILITY OF DERIVATIVES OF BLASCHKE PRODUCTS AND DIRICHLET-TYPE SPACES

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Question 1 (posed in [4]**).** Let B be a Blaschke product whose sequence of zeros $\{a_k\}_{k=1}^{\infty}$ satisfies that

$$\sum_{k=1}^{\infty}(1 - |a_k|)^{1/2} < \infty.$$

Does B' belong to H^p for some p, or to the Nevanlinna class?

For such a Blaschke product B, Théoremé IX of of [2] asserts that B' has finite radial limit at almost every point $\xi \in \partial\mathbb{D}$.

Question 2. For $0 < p < \infty$ let D_{p-1}^p be the space of analytic functions on the unit disc $\mathbb{D}$ such that

$$\int_{\mathbb{D}} |f'(z)|^p (1 - |z|)^{p-1}\, dA(z) < \infty,$$

where $dA(z) = \frac{dx\,dy}{\pi}$. The problem consists on describing the positive Borel measures μ on $\mathbb{D}$ such that $D_{p-1}^p \subset L^p(\mu)$ for $p > 2$. It is known that this embedding holds for $0 < p < 2$ if and only if $H^p \subset L^p(\mu)$, where H^p is the classical Hardy space [1]. See [3] and [5] for further information.

References

[1] P. Duren, Theory of H^p Spaces, Academic Press, New York-London 1970.

[2] O. Frostman, Sur le produits de Blaschke, Kungl. Fysiogr. Säsk. i Lund Förh. 12, 1942, 169–182.

[3] Girela, D., Peláez, J.: Carleson measures for spaces of Dirichlet type. Integral Equations and Operator Theory 55, n. 3, 415–427, (2006).

[4] J. A. Peláez, Sharp results on the integrability of the derivative of an interpolating Blaschke product, Forum Math. 20 (2008), no. 6, 1039–1054.

[5] J. A. Peláez, F. Peréz-González and J. Rättyä, Operator theoretic differences between Hardy spaces and Dirichlet-type spaces, J. Math. Anal. Appl. 418 (2014), no. 1, 387–402.

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